STABILITY ANALYSIS OF THIN-SHELL WORMHOLE LINEARIZED IN STRING THEORY

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Abstract- Using the cut-and-paste technique, we construct a thin-shell wormhole by surgically grafting together two copies of spacetimes of string inspired charged black hole solution. The total amount of exotic matter in the shell needed to sustain the wormhole is calculated and its dependence with the parameters is analyzed. The dynamical stability of the stringy thin-shell wormhole is analyzed by considering the linearized fluctuation around a static solution.

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1 INTRODUCTION

The study of traversable wormholes has received considerable attention from researchers for the past two and half decades. Over the last two decades, there has been considerable interest in the topic of thin-shall wormholes, solution of Einstein's field equations which act as tunnel from one region of spacetime to another, through which a traveler might freely pass [1-17]. It was found that these geometries, which act as tunnels from one region of spacetime to another, posses a peculiar property, namely exotic matter, involving a stress-energy tensor that violates the null energy condition [18-20]. In fact, traversable wormholes violate all of the pointwise energy conditions and averaged energy conditions [21]. As the violation of the energy conditions is a particularly problematic issue [22], it is useful to minimize the usage of exotic matter. The null energy and averaged null energy conditions are always violated for wormhole spacetimes. As it is difficult to deal with exotic matter, it is useful to minimize the usage of exotic matter.

Visser [2], the pioneer of thin-shell wormhole, has proposed a way, which is known as 'cut and paste' technique, of minimizing the usage of exotic matter to construct a wormhole in which the exotic matter is concentrated at the wormhole throat. In 'cut and paste' technique, the wormholes are theoretically constructed by cutting and pasting two manifolds to obtain geodesically complete a new one with a throat placed in the joining shell. By invoking the Darmois-Israel [23] formalism, the surface stresses of the exotic matter were determined. These thin-shell wormholes are extremely useful as one may apply a stability analysis for the dynamical cases, by choosing specific surface equations of state [24]. Recently, Eiroa and Romero [6] have extended the linearized stability analysis to Reissner-Nordström thin-shell geometries, and Lobo and Crawford [4] to wormholes with a cosmological constant. Visser and Poisson [3] have analyzed the stability of thin-shell wormhole constructed by joining the two Schwarzschild geometries. In this work, we study a new kind of thin-shell wormhole by surgically grafting two spacetimes of charged static black hole solution, which is often called the Gibbons-Maeda-Garfinkle-Horowitz-Strominger (GMGHS) black hole solutions. The linearized stability is analyzed under radial perturbation around a static solution. Throughout the paper we use $c = G = 1$.

The paper is organized as follows: In Sec. II, we construct a dynamic thin-shell wormhole by surgically grafting two spacetimes of GMGHS black hole. In Sec. III, we determine the total amount of exotic matter located at the thin-shell. In Sec. IV, we perform a detailed analysis of the stability under spherically symmetric perturbations around a static solution. Finally, conclusion of the results is given in Sec. V.

2 CONSTRUCTION OF THIN-SHELL WORMHOLE

The line element of spherically symmetric and static GMGHS black hole is given by [25]

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + h(r)\left(d\theta^2 + \sin^2\theta d\phi^2\right), \quad (1)$$

where

$$f(r) = \left(1 - \frac{2M}{r}\right), \quad (2)$$

$$h(r) = r^2\left(1 - \frac{Q^2 e^{2\phi_0}}{Mr}\right), \quad (3)$$

$$e^{-2\phi} = e^{-2\phi_0}\left(1 - \frac{Q^2 e^{2\phi_0}}{Mr}\right), \quad (4)$$

and

$$F = Q\sin\theta\, d\theta d\phi, \quad (5)$$

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where ϕ_0 is the asymptotic constant value of the dilaton field. The metric (1), describes a black hole of mass M and magnetic charge Q when the ratio Q/M is small.

The thin-shell wormhole is an interesting wormhole solution consists in applying the cut-and-past technique. We take two copies of GMGHS black hole solution (1), removing from each spacetime the four dimensional regions is described by

$$\Omega^{\pm} = \{ r^{\pm} \le a | a > r_b \}, \tag{6}$$

where a is a constant and r_b is the black hole event horizon, corresponding to the GMGHS black hole solution.

To avoid the presence of an event horizon, the important condition $a > r_b$ is applied. The removal of these two regions result in two manifolds, geodesically incomplete, with boundaries given by the following timelike hypersurfaces

$$\partial\Omega^{\pm} = \{ r^{\pm} = a | a > r_b \}. \tag{7}$$

Consider the junction surface $\partial\Omega$ as a timelike hypersurface defined by the parametric equation of the form $F\left(x^{\mu}(\xi^i)\right) = 0$. $\xi^i = (\tau, \theta, \phi)$ are the intrinsic coordinate on $\partial\Omega$, where τ is the proper time on the hypersurface. The unit normal 4-vector n^{μ} to $\partial\Omega$, is defined as

$$n_{\mu} = \pm \left| g^{\alpha\beta} \frac{\partial f}{\partial x^{\alpha}} \frac{\partial f}{\partial x^{\beta}} \right|^{-1/2} \cdot \frac{\partial f}{\partial x^{\mu}}, \tag{8}$$

with $n_{\mu} n^{\mu} = +1$ and $n_{\mu} e^{\mu}_{(i)} = 0$. The extrinsic curvature with the two sides of the shall are defined as

$$K_{ij}^{\pm} = -n_{\mu} \left(\frac{\partial^2 x^{\mu}}{\partial \xi^i \partial \xi^j} + \Gamma^{\mu\pm}_{\alpha\beta} \frac{\partial x^{\alpha}}{\partial \xi^i} \frac{\partial x^{\beta}}{\partial \xi^j} \right), \tag{9}$$

where the $\pm$ superscripts represent the exterior and interior spacetimes. Since the extrinsic curvature K_{ij} is not continuous across $\partial\Omega$, so the discontinuity in the extrinsic curvature is defined as $K_{ij} = K_{ij}^{+} - K_{ij}^{-}$. Using the Darmois-Israel formalism [9], at the junction interface $\partial\Omega$, the surface stress-energy tensor S^i_j is obtained by the Lanczos equations

$$S^i_j = -\frac{1}{8\pi} \left[k^i_j - \delta^i_j k^k_k \right], \tag{10}$$

where $\boldsymbol{k^i_j}$ is the discontinuity of the extrinsic curvatures across the interface $\partial\Omega$.

In terms of the surface energy density σ and the surface pressure p, the surface energy tensor may be written as $S^i_j = diag(-\sigma, p, p)$. The thin-shell equations which is commonly known as Einstein's field equations then become

$$\sigma = -\frac{1}{4\pi} K^{\theta}_{\theta}, \tag{11}$$

$$p = \frac{1}{8\pi} \left(K^{\tau}_{\tau} + K^{\theta}_{\theta} \right). \tag{12}$$

In the orthonormal basis $\left\{ e_{\hat{\tau}}, e_{\hat{\theta}}, e_{\hat{\phi}} \right\} \left\{ e_{\hat{\tau}} = e_{\tau}, e_{\theta} = (h(a))^{-1/2} e_{\theta}, e_{\hat{\varphi}} = \left[h(a) \sin^2\theta \right]^{-1/2} e_{\varphi} \right\}$ to the metric (1), we obtain

$$K^{\pm}_{\hat{\theta}\hat{\theta}} = K_{\hat{\varphi}\hat{\varphi}} = \pm \frac{h'(a)}{2h(a)} \sqrt{f(a) + \dot{a}^2}, \tag{13}$$

$$K^{\pm}_{\hat{\tau}\hat{\tau}} = \mp \frac{2\ddot{a} + f'(a)}{2\sqrt{f(a) + \dot{a}^2}}. \tag{14}$$

The components of the surface stress-energy can be deduced from Eqs. (11) and (12) as

$$\sigma = -\frac{1}{4\pi} \frac{h'(a)}{h(a)} \sqrt{f(a) + \dot{a}^2}, \tag{15}$$

$$p_{\theta} = p_{\phi} = p = \frac{1}{8\pi} \frac{h'(a)}{h(a)} \sqrt{f(a) + \dot{a}^2} + \frac{1}{8\pi} \frac{2\ddot{a} + f'(a)}{\sqrt{f(a) + \dot{a}^2}}. \tag{16}$$

According to the flaring out condition the area is minimal at the throat (then h(r) increases for r close to a and $h'(a) > 0$), implies that the energy-density σ is negative at the throat, so exotic matter is located there. The pressure p may be positive.

3 TOTAL AMOUNT OF EXOTIC MATTER IN THE SHELL

The total amount of exotic matter presents in the shell can be quantified following [26] by the integral

$$\Omega_{\sigma} = \int [\rho + p_r] \sqrt{-g}\, d^3x, \tag{17}$$

where g is the determinant of the metric tensor. We introduce a new radial coordinate $R = \pm(r - a)$ in Ω (for $\Omega^{\pm}$ respectively), one obtain

$$\Omega_{\sigma} = \int_0^{2\pi} \int_0^{\pi} \int_{-\alpha}^{\alpha} [\rho + p_r] \sqrt{-g}\, dR\, d\theta\, d\phi. \tag{18}$$

Since the shell is infinitely thin, the exotic matter does not exert any radial pressure, it only exerts tangential pressure and it is placed in the shell, so that $\rho = \delta(R)\sigma$ (δ is the Dirac delta function). We therefore obtain

$$\Omega_\sigma = \int_0^{2\pi}\int_0^{\pi} \rho\sqrt{-g}\ \big|_{r=a}\, d\theta\, d\phi = \int_0^{2\pi}\int_0^{\pi} \sigma\sqrt{-g}\ \big|_{r=a}\, d\theta\, d\phi = 4\pi\, h(a)\,\sigma(a). \qquad (19)$$

Now using Eqs. (3) and (15), we have

$$\Omega_\sigma = -2a\sqrt{\left(1-\frac{2M}{a}\right)\left(1-\frac{Q^2 e^{2\phi_0}}{2Ma}\right)}. \qquad (20)$$

Thus, the total amount of exotic matter needed is depend on black hole mass M, magnetic charge Q and asymptotic constant value of the dilaton field φ0. If the mass M and charge Q of the black hole are fixed, then the total amount of exotic matter is reduced by increasing the asymptotic constant value of the dilaton. Also if the magnetic charge of the black hole and the asymptotic constant value of the dilaton are kept constant, then the total amount of exotic matter is reduced by decreasing the mass of the black hole.

4 DYNAMIC STABILITY ANALYSIS OF THIN-SHELL WORMHOLE

The standard stability analysis method for thin-shell wormhole, based on the definition of a potential, is extended to our metric (1). From the Einstein's field equations, it is easy to check the energy conservation equation

$$\frac{d}{d\tau}(\sigma A) + p\frac{d}{d\tau}A = \left\{[h'(a)]^2 - 2h(a)h''(a)\right\}\times\frac{\dot{a}\sqrt{f(a)+\dot{a}^2}}{2h(a)}, \qquad (21)$$

where $A = 4\pi\, h(a)$ is the area of the wormhole throat. The first term in the left hand side of Eq. (23) represents the change in internal energy of the throat and the second one is the work done by the internal forces of the throat, while according to the Ref. [15], the term in the right-hand side represents a flux. The above equation then may be written in the form

$$\frac{d}{da}[\sigma h(a)] + p\frac{d}{da}[h(a)] = -\left\{[h'(a)]^2 - 2h(a)h''(a)\right\}\times\frac{\sigma}{2h'(a)}. \qquad (22)$$

When $[h'(a)]^2 - 2h(a)h''(a) = 0$, the flux term in the right-hand side of Eqs. (21) and (22) is zero and they take the form of simple conservation equations. From Eq. (22), we obtain

$$h(a)\sigma' + h'(a)(\sigma+p) + \left\{[h'(a)]^2 - 2h(a)h''(a)\right\}\frac{\sigma}{2h'(a)} = 0, \qquad (23)$$

where '′' denotes differentiation with respect to a. If we choose a particular equation of state, in the form of $p = p(\sigma)$, then we can formally integrate the conservation equation and obtain

$$\ln(a) = -\frac{1}{2}\int\frac{da}{\sigma+p}. \qquad (24)$$

This relationship may then be formally inverted to yield σ as a function of the wormhole radius, $\sigma = \sigma(a)$. Thus, rearranging the terms of Eq. (15), the dynamics of the wormhole throat is completely determined by a single equation $\dot{a}^2 + V(a) = 0$. Here the potential $V(a)$ is defined as

$$V(a) = f(a) - 16\pi^2\left[\frac{h(a)}{h'(a)}\sigma(a)\right]^2. \qquad (25)$$

To perform the linearized stability analysis, choice is to consider linearized fluctuations around an assumed static solution characterized by the constants a_0, σ_0 and p_0. Assuming this assumption, from Eqs. (15) and (16), we have

$$\sigma_0 = -\frac{1}{2\pi a_0}\frac{\left(1-\frac{Q^2e^{2\phi_0}}{2Ma_0}\right)\sqrt{1-\frac{2M}{a_0}}}{\left(1-\frac{Q^2e^{2\phi_0}}{Ma_0}\right)}. \qquad (26)$$

$$p_0 = \left[\frac{\left(1-\frac{Q^2e^{2\phi_0}}{2Ma_0}\right)\left(1-\frac{2M}{a_0}\right)+\frac{M}{a_0}\left(1-\frac{Q^2e^{2\phi_0}}{Ma_0}\right)}{\left(1-\frac{Q^2e^{2\phi_0}}{Ma_0}\right)\left(\sqrt{1-\frac{2M}{a_0}}\right)}\right]. \qquad (27)$$

A Taylor series expansion to second order in $a = a_0$ of the potential $V(a)$ around the static solution, yields

$$V(a) = V(a_0) + V'(a_0)[a-a_0] + \frac{1}{2}V''(a_0)[a-a_0]^2 + O[a-a_0]^3, \qquad (28)$$

The first derivative of the potential $V(a)$ can be obtained from Eq. (25) as

$$V'(a) = f'(a) - 32\pi^2\sigma(a)\frac{h(a)}{h'(a)}\times\left\{\left[1-\frac{h(a)h''(a)}{[h'(a)]^2}\right]\sigma(a)+\frac{h(a)}{h'(a)}\sigma'(a)\right\}. \qquad (29)$$

Now using Eq. (23), the above equation can be written as

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$$V'(a)=f'(a)+16\pi^2\sigma(a)\frac{h(a)}{h'(a)}\left[\sigma(a)+2p(a)\right]. \tag{30}$$

The second derivative of the potential $V(a)$ is

$$V''(a)=f''(a)+16\pi^2\times\left\{\left[\frac{h(a)}{h'(a)}\sigma'(a)+\left(1-\frac{h(a)h''(a)}{\left[h'(a)\right]^2}\right)\sigma(a)\right]\left[\sigma(a)+2p(a)\right]+\frac{h(a)}{h'(a)}\sigma(a)\left[\sigma'(a)+2p'(a)\right]\right\}. \tag{31}$$

We now define a parameter β by the relation

$$\beta^2(\sigma)\equiv\frac{\partial p}{\partial\sigma}\Big|_{\sigma}\,. \tag{32}$$

The physical interpretations of β under normal circumstances interpret as the subluminal sound speed. Here we simply consider β to be a useful parameter related to the equation of state. By this definition, we have

$$\sigma'(a)+2p'(a)=\sigma'(a)(1+\beta^2)\,. \tag{33}$$

Using Eqs. (23) and (33), we can rewrite $V''(a)$ as follows

$$V''(a)=f''(a)-8\pi^2\times\left\{\left[\sigma(a)+2p(a)\right]^2+2\sigma(a)\left[\left(\frac{3}{2}-\frac{h(a)h''(a)}{\left[h'(a)\right]^2}\right)\sigma(a)+p(a)\right]\left(1+2\beta^2\right)\right\}. \tag{34}$$

Since we are linearizing around a static solution at $a=a_0$, we must have $V(a_0)$ and $V'(a_0)$ are equal to zero. To leading order, therefore, $V(a)=\frac{1}{2}V''(a_0)\left[a-a_0\right]^2$. The configuration will then be in stable equilibrium if $V''(a_0)>0$. Now

$$V''(a_0)=-2a_0^{-2}\left[\frac{2M}{a_0}+\frac{M^2}{a_0^2\left(1-\frac{2M}{a_0}\right)}+\left(1+2\beta_0^2\right)\left\{\frac{\left(1-\frac{3M}{a_0}\right)\left[1+\frac{1}{2}\left(\frac{Q^2e^{2\phi_0}}{Ma_0}\right)^2-\frac{Q^2e^{2\phi_0}}{Ma_0}\right]+\frac{1}{2}\frac{Q^2e^{2\phi_0}}{a_0^2}}{\left(1-\frac{Q^2e^{2\phi_0}}{Ma_0}\right)^2}\right\}\right]. \tag{35}$$

At this order of approximation, the equation of motion of the throat is

$$\dot{a}^2=-\frac{1}{2}V''(a_0)\left[a-a_0\right]^2+O\left[a-a_0\right]^3. \tag{36}$$

From Eq. (34) and considering $V''(a_0)=0$, one can find the expression for β_0 which is given by

$$\beta_0^2=-\frac{1}{2}-\frac{M}{a_0}\times\frac{\left(1-\frac{3M}{2a_0}\right)}{\left(1-\frac{2M}{a_0}\right)}\times\frac{\left(1-\frac{Q^2e^{2\phi_0}}{Ma_0}\right)^2}{\left[\left(1-\frac{3M}{a_0}\right)\left\{1+\frac{1}{2}\left(\frac{Q^2e^{2\phi_0}}{Ma_0}\right)^2-\frac{Q^2e^{2\phi_0}}{Ma_0}\right\}+\frac{1}{2}\frac{Q^2e^{2\phi_0}}{a_0^2}\right]}, \tag{37}$$

has a simple vertical asymptote to the right of the asymptote,

$$\left(1-\frac{2M}{a_0}\right)\left[\left(1-\frac{3M}{a_0}\right)\left\{1+\frac{1}{2}\left(\frac{Q^2e^{2\phi_0}}{Ma_0}\right)^2-\frac{Q^2e^{2\phi_0}}{Ma_0}\right\}+\frac{1}{2}\frac{Q^2e^{2\phi_0}}{a_0^2}\right]>0. \tag{38}$$

Returning to the inequality $V''(a_0)>0$, at the throat $a=a_0$, we therefore have

$$\beta_0^2<-\frac{1}{2}-\frac{M}{a_0}\times\frac{\left(1-\frac{3M}{2a_0}\right)}{\left(1-\frac{2M}{a_0}\right)}\times\frac{\left(1-\frac{Q^2e^{2\phi_0}}{Ma_0}\right)^2}{\left[\left(1-\frac{3M}{a_0}\right)\left\{1+\frac{1}{2}\left(\frac{Q^2e^{2\phi_0}}{Ma_0}\right)^2-\frac{Q^2e^{2\phi_0}}{Ma_0}\right\}+\frac{1}{2}\frac{Q^2e^{2\phi_0}}{a_0^2}\right]}. \tag{39}$$

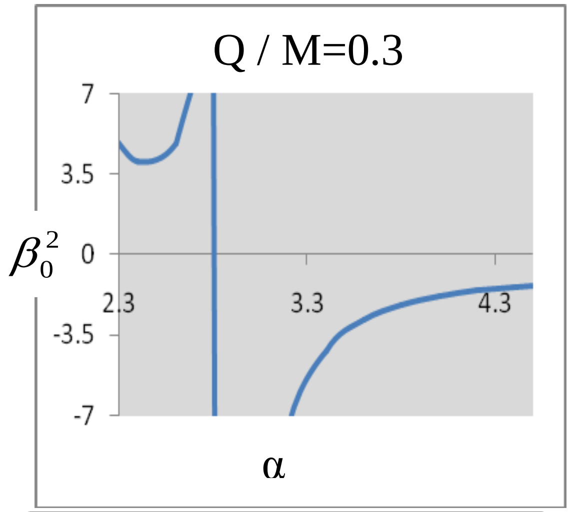
So to the right of the asymptote, the stability region of the wormhole is below the graph of Eq. (37), which is shown in Fig. 1.

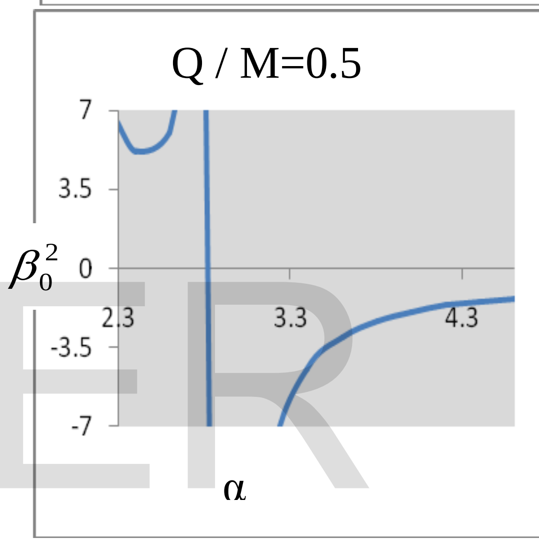
To the left of the asymptote, the sign of the inequality in Eq. (39) is reversed and one can obtain at $a=a_0$

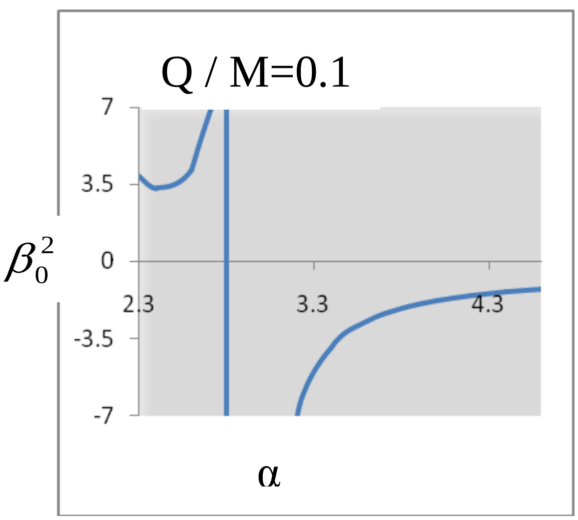
$$\beta_0^2 > -\frac{1}{2} - \frac{M}{a_0} \times \frac{\left(1-\frac{3M}{2a_0}\right)}{\left(1-\frac{2M}{a_0}\right)} \times \frac{\left(1-\frac{Q^2 e^{2\phi_0}}{Ma_0}\right)^2}{\left[\left(1-\frac{3M}{a_0}\right)\left\{1+\frac{1}{2}\left(\frac{Q^2 e^{2\phi_0}}{Ma_0}\right)^2 - \frac{Q^2 e^{2\phi_0}}{Ma_0}\right\} + \frac{1}{2}\frac{Q^2 e^{2\phi_0}}{a_0^2}\right]} . \quad (40)$$

So to the left of the asymptote, the stability region is above the graph.

Fig. 1 shows typical regions of stability using arbitrary values of the various parameters: $\phi_0 = 0.1$, $\frac{Q}{M} = 0.1$, $\frac{Q}{M} = 0.3$, $\frac{Q}{M} = 0.5$, and $\frac{Q}{M} = 0.7$. It is observed that the regions above the curves on the left and below the curves on the right are stable. The sign change is determined by inequality (38). For the values $\frac{Q}{M} = 0.7$ and more, the stable region above the left of the curve is not found. This is an excellent agreement with GMGHS black hole condition.







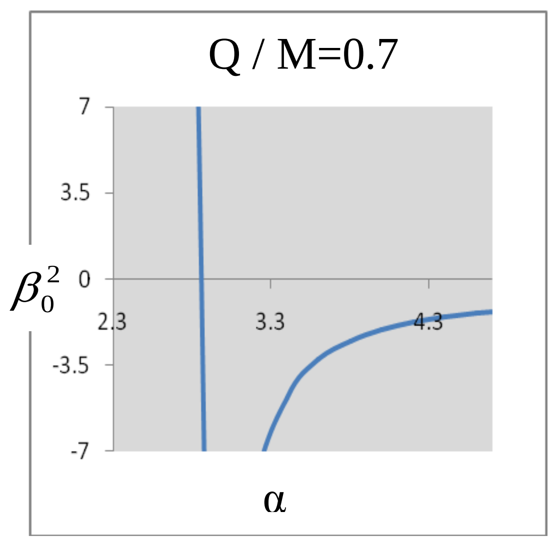


Fig.1: We have defined $\alpha = \frac{a_0}{M}$. Here we have plotted α versus β_0^2. Stability regions for the thin-shell wormhole with $\phi_0 = 0.1$ and different values of the scalar charge Q.

5 CONCLUSION

In this paper, we have constructed a charged thin-shell wormhole in dilaton gravity by surgically grafting two GMGHS black hole spacetimes. The surface energy density and tangential surface pressure on the shell is determined, the surface energy density is negative which ensures one of the most important criteria to construct a thin-shell wormhole. The exotic matter is localized at the thin-shell and found that the total amount of exotic matter needed is depend on black hole mass M, magnetic charge Q and asymptotic constant value of the dilaton field ϕ_0 and we conclude that less exotic matter is needed when magnetic charge and asymptotic constant value of the dilaton field of the black hole are increased and/or the mass of the black hole is decreased.

We have analyzed the dynamical stability of the thin-shell, considering the linearized radial perturbation around the static solution at $a = a_0$. The stability analysis concentrated on the parameter β, which is a useful parameter and related to the equation of state. The parameter β_0 normally interpreted as the speed of sound and the order of magnitude is same as the speed of sound. The region of stability is obtained in terms of the mass of the wormhole *M*, the radius of the wormhole throat a_0 and a parameter β_0. The region of stability lies β_0^2 - α plane. It is observed that for low values of $\frac{Q}{M}$, the stable region is significant.

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